Estimation of Social Distancing Through the Probabilistic Weiss Equation: It is the Wind Velocity a Relevant Factor?

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***Abstract*—From the probabilistic Weiss equation, various relations involving the distance, wind velocity and number of people both healthy and infected, the critic distances that might be critic to transmit any virus strain, are calculated. The present approach considers as main criterion that the outdoor infection is a random event that depends of a plethora of variables and free parameters. We project the present theoretical proposal to the current Corona Virus Disease 2019 pandemic by which is established that people must keep a social distance ranging between 1.5m and 2.0m in order to avoid contact with aerosol. In this paper stochastic and deterministic equations are proposed as well as hybrid relationships that would explain facts in the action of outdoor infection. In this manner virus can be transmited in a radius of 2m for a wind velocity of 10m/s.**

***Index Terms*—Covid-19, Weiss probability, epidemiology.**

I. Introduction

The ongoing pandemic Corona Virus Disease 2019 (Covid-19) has arrived in a large number of countries at the beginning of 2020 [1][2]. The rapidity of propagation is reflected at the peaks of infections as seen along the subsequent months at the country statistics. In fact, so far not any plan of vaccine is in hands, clearly one must expects successive peaks and waves [3]. In addition public health operators are expected to attenuate the propagation of Covid-19 in vulnerable population [4]. and is also a substantial cause for economic tumble in developing countries [5]. As consequence of disease arrival, a large number of economies might to delay their expected developing as well as it is expected other consequences at next 2021 [6].

Apart the public health policies, Covid-19 has manifested various outbreaks as well as the first and second wave exhibiting still its powerfulness until the date that a vaccine is applied [7]. As a fast response to the fast spread of virus, it were imposed social restrictions whose main target is to avoid massive infections. In addition, the social distancing was imposed among people with values that are ranging between 1.5m to 2m. A serious problem might to constitute the scenario of outdoor infection by which one finds various variables that cannot be controllable such as the wind velocity and other kind of scenarios that can contain for large times the expelled aerosol [8][9]. Therefore, this paper tries to ask the question: **What is the critic distance by which one healthy people can be infected due to the contact of "N" droplets containing the Covid-19?** To answer this question is needed to understand the problem of dynamics of aerosol indoor that might be also perceived as a stochastic phenomenon more than an exact or deterministic problem. In this manner it is necessary to establish the contribution of each one of the following points to the action of outdoor infection (by which is assumed that lockdown is done and people can stay outdoor in both scenarios either with or without):

- Wind velocity,
- Probability of finding infected people outdoor in a random time t,
- Periods of accumulation of people,
- Dynamical of aerosol,
- Robustness of droplets,
- Random number of infected speech droplets expelled,

In this manner, it is suitable to propose a fully stochastic model based essentially in the assumption: **How long would travel an infected speech droplet prior to dehydration?**. An intuitive toy model of infection probability would be modeled by a simple Gaussian profile:

$$\mathcal{P}(x) = p_0 \mathrm{Exp}\left[-\left(\frac{x-x_0}{\Delta x}\right)^2\right] \tag{1}$$

with $x-0$ the position of infected people. Thus, while one is close to x_0 the probability becomes higher. The findings of Stadnytskyi $et.al$ [10] revealed that in confined spaces and conditions of humidity 27% and $23C^0$, airborne can remain for more that 8 minutes. In an intuitive manner one can anticipate that in outdoor conditions determinism might not be appropriate to model a scenario where there is a noted confluence of random variables. In second section, the theoretical machinery is built. Firstly a naive probabilistic model is provided. With this the Weiss theory is employed to calculate the critic distribution of distances of infection. In third section, the computational simulations are done. With this, the results are discussed. Finally in fourth section, the conclusion of paper is drawn.

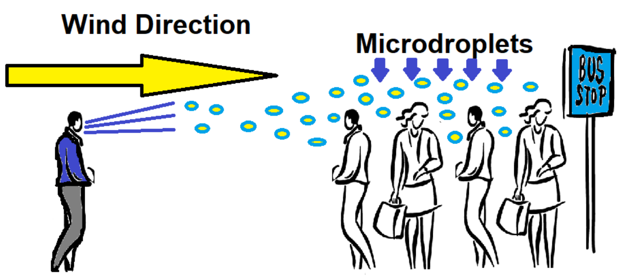

Fig. 1. Sketch of public infection: One single infected can send thousands of microdroplets in loud speech. Depending on the wind velocity the spatial dispersion of the microdroplets can span long radii affecting healthy people as sketched for example the case of a Bus stop.

II. The wind velocity as cause of airborne droplets dispersion

Consider airborne speech droplets. It is assumed that all of them are transporting virus. In an outdoor scenario (streets for example) one can establish the following scenario:

- Droplets have a negligible mass,
- Droplets have same velocity of wind,
- The number of infected droplets is proportional to the number of people infected,
- The number of infected people in streets is random,
- Thus, the number of infected droplets is also random,
- Humidity and temperature might have no any effect on the lifetime of droplets.

In this manner it is proposed the well-defined scenarios by which one or more infected people are expelling infected droplets in an environment of healthy people at a certain time [11]. Consider a time t in any city where is known that the wind velocity is v. In this manner, any droplets assumed to have a spherical shape has a volume and mass that do not offers resistance to air, therefore droplets move at same wind direction. Although one can apply the methodology of Reynolds number equation, the present methodology opts by one that neglects aerodynamics. Instead of that, the probabilistic intuition is employed. Experience tell us that while a single person is talking loud it is logic to expect that is expelled a large amount of microdroplets. Thus the change at time of net number of expelled $\mathbf{n}$ microdroplets traveling due to a wind velocity at an arbitrary distance s is assumed to be proportional to the distribution of infected people [12] $\mathbf{D}$ times the number $\mathbf{N}$ of expelled microdroplets for them times the change of their velocity Δv. This relationship can be defined at the position s and time t as:

$$\frac{\Delta \mathbf{n}(s)}{\Delta t} = \mathbf{N}(s)\mathbf{D}(s,t)\Delta v. \tag{2}$$

The interpretation of Eq.(2) is in part illustrated in Fig.1. Consider a single person (that can also be extended to various people $\mathbf{D}$) expelling $\mathbf{N}$ microdroplets. The wind velocity plays the role as medium that keeps the microdroplets sustained at a distance $s$until their dehydration. It should be noted that this distance can reach a random number of healthy people (as sketched at Fig.1 as for example a Bus stop). From Eq.2 one arrived to:

$$\frac{\Delta \mathbf{n}(s)}{\Delta t \Delta v} = \mathbf{N}(s)\mathbf{D}(s,t), \tag{3}$$

by which one can estimate the change of $\mathbf{n}(s)$ with respect to the displaced distance and that can be written as:

$$\frac{\Delta \mathbf{n}(s)}{\Delta s} = \mathbf{N}(s)\mathbf{D}(s,t). \tag{4}$$

From this one can carry out the integration as follows:

$$\mathbf{n}(x,t) = \int_{s_A}^{s_B} ds \mathbf{N}(s,x)\mathbf{D}(s,t) \tag{5}$$

with $s_A < x < s_B$. In addition one can incorporate the droplet velocity at the integration limits, so that Eq.52) can b finally be written as:

$$\mathbf{n}(x,t,v_A,v_B) = \int_{v_A t}^{v_B t} ds \mathbf{N}(x,s)\mathbf{D}(s,t). \tag{6}$$

Here it is plausible to establish that v_A can be considered as the wind velocity. With respect to the upper limit of integration, experience tell us that because hydration $v_B \approx 0$. In other words the microdroplet has clearly a well-defined lifetime so that its range of displacement is entirely governed by its physical composition. Here it should be also noted that in outdoor conditions, the microdroplet has a direct dependence with respect to:

- Humidity,
- Outdoor temperature,
- Environment conditions,
- Solar light,

that would have a strong influence with respect to the final velocity of microdroplet (an complete and accurate analysis of this goes beyond the scope of this investigation). Therefore with all this then one gets that:

$$\mathbf{n}(x,t,v_A) = \int_{v_A t}^{0} ds \mathbf{N}(x,s)\mathbf{D}(s,t). \tag{7}$$

In Fig.2 are displayed simple applications of Eq.7. In up panel the number of droplets with respect to time is plotted. In down panel the case where $\mathbf{n}$ is done as function of their velocity. For this illustrations the following integrations were used: $500 \int_{5t}^{5qt} ds \mathrm{Exp}[-(\frac{s-5qt-5q}{10q})^2]\ \mathrm{Exp}[-(\frac{s-t-10}{10})]$ (up panel) and $650 \int_{0.1}^{3qv} ds \mathrm{Exp}[-(\frac{s-3qv-5}{10})^2]\ \mathrm{Exp}[-(\frac{s-12}{10})]$ (down panel). In both cases curves (where time and velocity are done in arbitrary units) exhibit a peak with a subsequent fall in accordance to common sense and the criterion of dehydration of all microdroplets in free space.

III. Encompassing with the Weiss Equation

In order to engage Eq.(7) with the theory of Weiss [13], it is needed to establish the following definitions:

- Q objects → Q infected microdroplets,
- N cells → N people,
- M the times by which is expected the arrived of Q microdroplets,

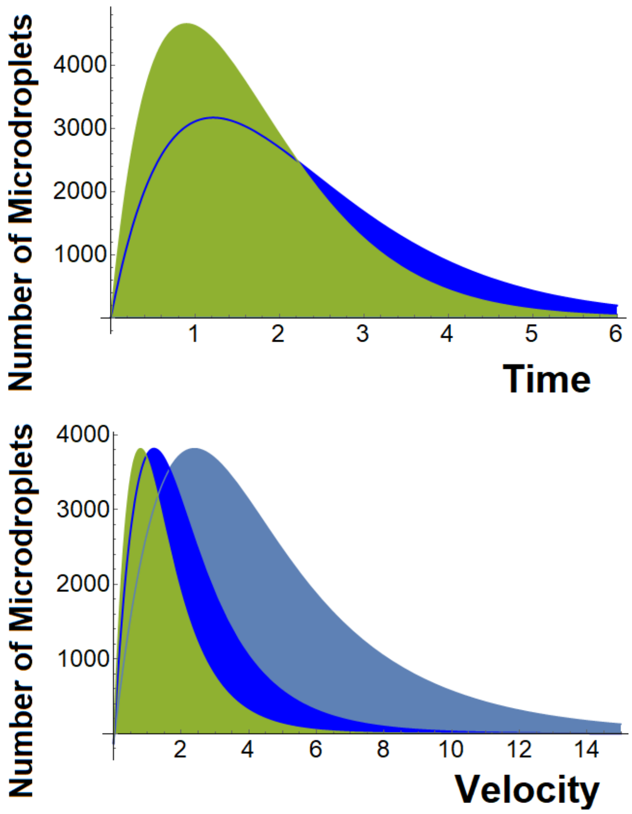

Fig. 2. Illustration of Eq.7 for the cases where **N** and **D** are represented by a Gaussian and negative exponential. Up and down panel were done with a Gaussian width of $10\times q$ (with q=1,2,3) and 10 respectively.

- λ a random number that encompasses the aleatory character per each action.

In this manner the Weiss equation is formulated as follows: Consider that Q objects are sent to N cells with the restriction that only one object must fall to only one cell. When this action is repeated M times until N and QM becomes large numbers, thus is plausible to define the probability for finding an unoccupied cell given by:

$$P(M) = \frac{N\text{Log}N}{QM}. \tag{8}$$

Thus, one can interpret that the unoccupied cell is analogue to state that a single people was not reached by the microdroplets. Therefore, one can take advantage of Eq.(8) to introduce Eq.(7) on it. For this end one can define $\omega = 1/M$ as the frequency of the arrived of infected microdroplets on the aggregation of healthy people. With this the probability reads

$$P(\omega) = \frac{\omega N\text{Log}N}{Q}. \tag{9}$$

With this, one can rewrite the probability as being proportional to a logarithm in the sense that

$$P(\omega) = \frac{N}{Q}\text{Log}N^{\omega}. \tag{10}$$

One can see that there is a "closed-form" route to extract the variable N that denotes the number of cells or healthy people. Therefore one has that:

$$P(\omega)\frac{Q}{N} = \text{Log}N^{\omega}. \tag{11}$$

One can note that the variables of both left and right side can be perceived as pure random numbers at the sense that the precise knowledge of their values cannot be known exactly at a time t [14]. Thus one can establish that $R(\omega, Q, N) = \lambda$, with a λ a random number. On the other side a kind of separability can be imposed and given by: $R(\omega, Q, N)$=$R_A(Q, N)R_B(\omega)$. From this one can arrive to:

$$R_A(Q, N) = \frac{\text{Log}N^{\omega}}{R_B(\omega)} = \lambda, \tag{12}$$

thus the number of cells N can be extracted in a straightforward manner

$$N(\omega) = \left(\text{Exp}[\lambda R_B(\omega)]\right)^{\frac{1}{\omega}}. \tag{13}$$

In order to illustrate Eq.(13) one needs a concrete value of $R_B(\omega)$. For the sake of simplicity it is represented by (1+Sin[ω])/2. In Fig.3 Eq.13 is illustrated through the following function: 5.75 (Exp[q$\times$0.25$\times$(1+Sin[q$\times\omega$])])$^{1/\omega}$. Under the assumption that there is 20 people that are receiving microdroplets in successive times, one can expect that whereas the frequency increases one can see that at least a 50% of them still are under risk of receiving microdroplets in a random manner [15]. From Eq.13 it is suitable to introduce the time as independent variable. In this manner one gets that:

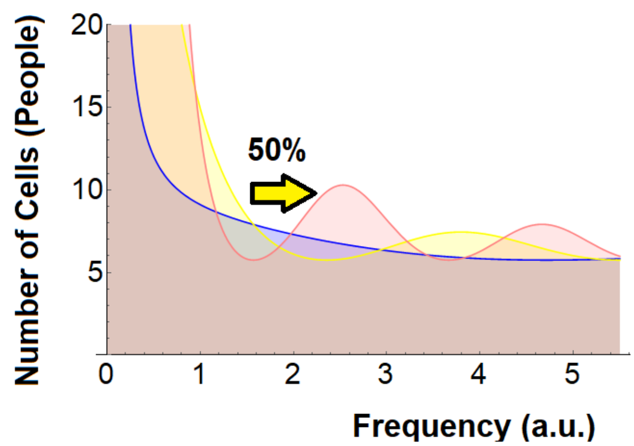

Fig. 3. Illustration of Eq.13 by using the 1/2(1+Sin(ω)) exhibiting that a 50% of healthy people stays in latent risk due to the arrival of microdroplets.

$$N\left(\frac{1}{t}\right) = \left(\text{Exp}[\lambda R_B(\frac{1}{t})]\right)^{t} \tag{14}$$

$$\approx \left(1 + \lambda R_B(\frac{1}{t})\right)^{t}, \tag{15}$$

by which any model for $R_B(\frac{1}{t})$ can be applied to extract the number of people under risk to be infected by infected microdroplets.

A. Semi Stochastic Weiss Model

For this approach one can employ a different approximation for $R_B(\frac{1}{t})$ such as $\text{Tan}\frac{1}{t}$ by which it is assumed that the microdroplets are traveling under same velocity as wind does without to abandon their physics laws [16][17]. Thus one can apply $t = \frac{x}{v}$, so the one arrives to the following definitions.

$$N\left(\frac{x}{v}\right) \approx \left(1 + \lambda\text{Sin}(\frac{v}{x})\right)^{\frac{x}{v}}, \tag{16}$$

$$N\left(\frac{x}{v}\right) \approx \left(1 + \lambda\text{Cos}(\frac{v}{x})\right)^{\frac{x}{v}}, \tag{17}$$

In Fig.4 both Eq.16 (up panel) and Eq.17 (down panel) seen as an normalized probability of risk as function of distance (in meters) are plotted. All plots Fig.1, Fig.2, Fig.3 and Fig.4 were done with Wolfram [18]. In up panel wind velocities of 5 m/s and 10 m/s were considered. It is interesting to see that black arrows are indicating the established "social distances" both 1m and 2m by the which the probabilities turns out to be small. In down panel same curves with same wind velocities indicating the "social distance" of around 1m. The following formula $0.83(1 + (q \times 0.5)\ [\text{Sin}, \text{Cos}](\frac{q\times 5}{x}))^{\frac{x}{q\times 5}}$.

B. Full Formulation

The combination of Eq.7 and Eq.8 can yield a complete picture of action of outdoor infection by microdroplets. If one assumes that the quantity Q is actually the number of microdroplets that are traveling along the air guided by the velocity of wind as well as following its direction, then one can write down a full formulation as follows:

$$P(M) = \frac{N\text{Log}N}{M\mathbf{n}} = \frac{N\text{Log}N}{M\int_{v_A t}^{0} ds\mathbf{N}(x,s)\mathbf{D}(s,t)} \tag{18}$$

where $\mathbf{N}(x,s) \neq N$. Indeed, by knowing that $\frac{1}{M} = \omega$ then this hybrid formulation can be expressed as a function of the times that a random number of infected microdroplets moves through the wind. Consider that N depends on time at the form of a polynomial as $N(t) = N(\frac{1}{\omega}) = \sum_J \frac{1}{J!}C_J(\frac{1}{\omega})^J$ with C_J random numbers, then under this approach one has that:

$$P(\omega) = \frac{N\text{Log}N}{\int_{v_A t}^{0} ds\mathbf{N}(x,s)\mathbf{D}(s,t)} = \frac{\omega\sum_J \frac{1}{J!}C_J(\frac{1}{\omega})^J \text{Log}\left[\sum_J \frac{1}{J!}C_J(\frac{1}{\omega})^J\right]}{\int_{v_A t}^{0} ds\mathbf{N}(x,s)\mathbf{D}(s,t)} \tag{19}$$

exhibiting its linearity with respect to frequency ω from the fact that there is a time dependence of people N that can be under risk. The integration at the denominator is also a nonlinear ingredient that might also be seen as a stochastic factor due essentially to the unpredictability of wind velocity. For a certain range of frequencies it is plausible to define the total probability as the integration of Eq.19 along the allowed values of ω and this operation can be written in a straight forward manner as:

$$\mathbf{P} = \int P(\omega)d\omega = \int \frac{\omega\sum_J \frac{1}{J!}C_J(\frac{1}{\omega})^J \text{Log}\left[\sum_J \frac{1}{J!}C_J(\frac{1}{\omega})^J\right]}{\int_{v_A t}^{0} ds\mathbf{N}(x,s)\mathbf{D}(s,t)} d\omega, \tag{20}$$

with $\mathbf{P} \approx \omega^{1-J}\ \text{Log}(\omega^{-J})$ perceived as a random number respect to the frequency of arrival of infected microdroplets.

IV. Conclusion

In this paper, a stochastic formulation of the problem of random outdoor infection due to the presence of infected microdroplets that are suspected to be traveling together with wind was presented. Although most of the independent variables treated in this paper belong to a stochastic territory, closed-form equations have been coherently derived. Therefore the probability of risk to receive infected microdroplets was plotted. Interestingly the resulting "social distances" of 1m. and 2m, as established by the health operators to avoid virus propagation have been consistently estimated. Wind velocities of order of 5m/s and 10m/s were employed in the computa-

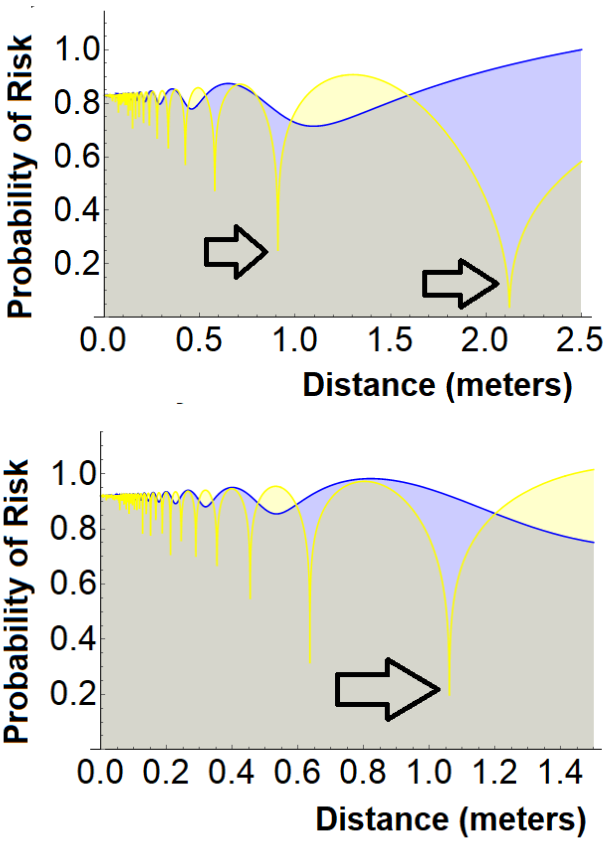

Fig. 4. Plots of Eq.16 and Eq.17 in the form of normalized probability of risk as function of distance of microdroplets assumed to be active before of dehydration.

tional analysis. These velocities would correspond to street's velocities by which one can find in Bus stop where is seen people accumulation. Clearly more analysis should be done in order to estimate the risk in urban as well as Peri-urban areas [19], particularly in all those places where wind velocities is high in those times where people is in transit and creates accumulations.

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