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## **CONTROL BY STATE SPACE AND FREQUENCY RESPONSE IN A DAMPED TRANSFEMORAL PROSTHESIS**

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***Abstract***—The present work is an applied research, that approaches the modeling and simulation of the control system by the frequency response method and the state space method in a transfemoral prosthesis with damping in the knee for applications in disabled patients. The research presents an alternative to improve the degree of independence for a person with an approximate height of  $1.70\text{m} \pm 2\text{cm}$  and a weight of  $70\text{kg} \pm 2\text{kg}$ .

***Keywords***-Control system, equations of state, frequency response, disability, prosthesis.

## I. INTRODUCTION

Amputation is appreciated as a public health inconvenience and an anatomical insufficiency that reduces the functional capacity of the person, changing their role in society and, in the same way, creates a personal, family and psychological impact. (Farro et al., 2012) [1].

In Peru, according to the Second National Specialized Survey on Disability and indicates that 2.9% of the national population expressed having at least one mobility and / or ability disability, this is around one million people.

The research sets three specific objectives: simulate the behavior of a knee prototype through a structural analysis using SolidWork software, use the state space theory and frequency response to control a prosthesis prototype, simulate the results using Matlab software.

## II. MATERIALES Y METODOS

The materials and methods is focused on being able to design a simulation through the use of an application, in order to test and analyze the knee prosthesis.

The following figure shows a finished model of the assembly of the proposed prosthesis, which was made using Solidworks software.

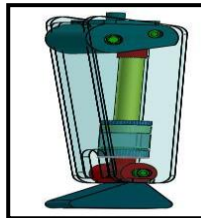


Fig. 1. Ensamble del sistema que se evalúa en Solidworks.

### A. Automatic control system

An automatic control system is a grouping of physical elements related or connected to each other, so that they regulate or command their performance by themselves, that is, without depending on external agents, which include the human factor, that can modify their operation [2].

**Transfer function:** To establish the response of a component as a function of time, known signals are used at the input of the system and the signals obtained at the output are evaluated. This temporal analysis can be studied using the transfer function or frequency response [3].

**State space representation:** The dynamic response of a system is represented by a set of state variables. This requires a minimum number of necessary and sufficient state variables that allow the dynamic description of the system. [3].

**State variables:** The state variables of a dynamic system are those that form the smallest set of variables that define a dynamic system. State variables should not necessarily be measurable or observable physical quantities [3].

**Frequency response method:** Frequency response methods are the most effective in conventional control theory, and are also necessary for robust control theory. The Nyquist stability criterion allows us to examine the relative and absolute stability of linear closed-loop systems from the knowledge of their open-loop frequency characteristics.

An advantage of the frequency response method is that frequency response testing is typically simple and accurate with the use of sinusoidal signal generators and accurate measurement equipment.

### *B. Damping System*

Sizing and functionality studies are prepared, such as the maximum and minimum piston stroke.

**Shock Absorber Shirt:** It is the part in the shock absorber where the other elements will be attached. See Fig. 2.a.

**Shock Plunger:** This element is incorporated into the upper pressure support. See Fig. 2.b.

**Support foot:** This element serves as a support in contact with the ground. See Fig. 2.c.

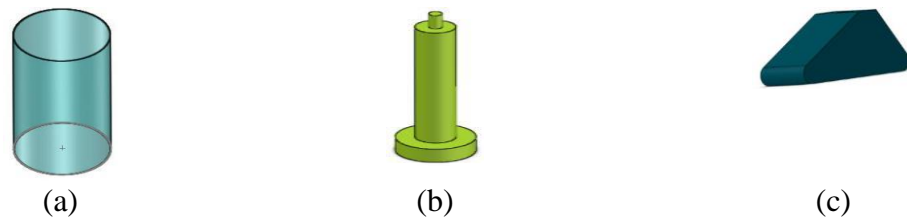


Fig. 2. Solidwork designed parts (a) Shock absorber sleeve. (b) Shock Piston. (c) Support foot.

### C. Equivalent dynamics of the patient

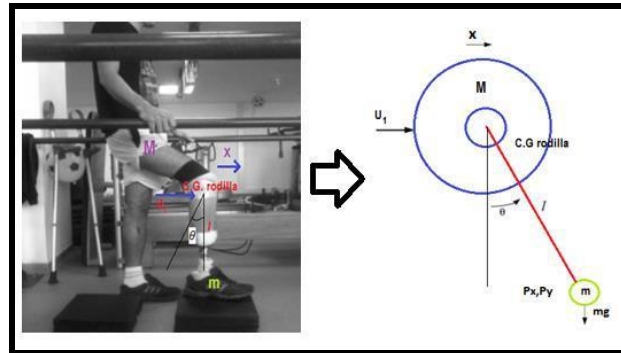


Fig. 3. Equivalence of the patient for modeling.

Where:

$M$  = mass of the patient

$m$  = foot mass

$l$  = length at center of mass

$U_1$  = locomotive force of the person

$x$  = This variable is the displacement at the moment of walking on the  $x$  axis.

$\dot{x}$  = velocity about the  $x$  axis.

$\theta$  = angle of the pendulum with respect to the vertical. This variable is the angle of rotation of the knee with respect to the vertical.

The disk represents the center of gravity of the mass of the person.

The under-actuated overhead crane model can be fundamentally accepted as a rigid body whose displacement is limited to two degrees of freedom, which are  $x$  and  $\theta$  [4].

The equations used are shown in (1):

$$(1) \quad \begin{aligned} L &= \sum E_c - \sum E_p \\ I &= \int_{t_1}^{t_2} L(q_i, \dot{q}_i) dt \end{aligned}$$

To find the kinetic energy that the car of mass  $m$  has, it is done by (2).

$$(2) \quad E_{c_g} = \frac{1}{2} M_g \dot{x}^2$$

Through (3) the kinetic energy of the pendulum is obtained:

$$(3) \quad E_{c_p} = \frac{1}{2} m(\dot{p}_x^2 + \dot{p}_y^2)$$

Defining the link of length  $l$  and the angle  $\theta$  to find the position of the pendulum  $p_x$  and  $p_y$ , using (5):

$$(5) \quad \begin{aligned} p_x &= x - l \sin \theta \\ p_y &= -l \cos \theta \end{aligned}$$

Finding the velocities of using (5).

$$(6) \quad \begin{aligned} \dot{p}_x &= \frac{d}{dt}(x - l \sin \theta) = \dot{x} - l \dot{\theta} \cos \theta \\ \dot{p}_y &= \frac{d}{dt}(-l \cos \theta) = l \dot{\theta} \sin \theta \end{aligned}$$

Squaring and adding (6), we obtain (7).

$$\begin{aligned} p_x^2 + p_y^2 &= \dot{x}^2 - 2l\dot{\theta}\dot{x}\cos\theta + l^2\dot{\theta}^2\cos^2\theta + l^2\dot{\theta}^2\sin^2\theta \\ p_x^2 + p_y^2 &= \dot{x}^2 - 2l\dot{\theta}\dot{x}\cos\theta + l^2\dot{\theta}^2 \end{aligned} \quad (7)$$

Replacing in (2) and (3) in (8), we obtain (9).

$$(8) \quad \begin{aligned} E_c &= E_{c_g} + E_{c_p} \\ (9) &= \frac{1}{2}M_g\dot{x}^2 + \frac{1}{2}m(\dot{x}^2 - 2l\dot{\theta}\dot{x}\cos\theta + l^2\dot{\theta}^2) \end{aligned}$$

The potential energy of the system is due only to gravity, but since gravity does not intervene in the movement of the car, the resultant is zero. Considering potential energy of (10), the Lagrangian shown in (11).

$$(10) \quad p = -mgl \sin \theta = mgl(1 - \cos \theta)$$

$$L = \frac{1}{2}(M_g + m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - ml\dot{\theta}\dot{x}\cos\theta + mgl\cos\theta$$

Applying the Euler-Lagrange equations [5] to (12), is obtained (13).

$$(12) \quad \begin{aligned} \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} &= 0 \\ x: \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} &= u_1 \\ \theta: \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} &= 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial \dot{x}} &= \frac{1}{2}M_g \frac{\partial}{\partial \dot{x}}(\dot{x}^2) + \frac{1}{2}m \frac{\partial}{\partial \dot{x}}(\dot{x}^2) - ml\dot{\theta}\cos\theta \frac{\partial}{\partial \dot{x}}(\dot{x}) \\ (13) & \\ &= (M_g + m)\dot{x} - ml\dot{\theta}\cos\theta \end{aligned}$$

Finally, deriving (13), the equations of motion of the system (14) are obtained.

$$(14) \quad \begin{aligned} (M_g + m)\ddot{x} - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta &= u_1 \\ -ml\dot{x}\cos\theta + ml^2\ddot{\theta} + mgl\sin\theta &= 0 \end{aligned}$$

Considering friction, we will redefine the equations of motion (15).

$$(M_g + m)\ddot{x} - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta + b\dot{x} = u_1 \quad (15)$$

**Transfer Function:** The transfer function shown in (16).

$$\frac{\Phi(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgI}{q}}$$

$$q = [(M + m)(I + ml^2) - (ml)^2]$$

**State Space Model:** The model in the corresponding state space is shown in the equation (17).

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-b}{M} & \frac{-gm}{M} & \frac{-b}{lM} \\ 0 & 0 & 1 & 0 \\ 0 & \frac{-b}{lM} & \frac{-g(M+m)}{lM} & \frac{-b(M+m)}{ml^2M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} u$$

$$(17) \quad y(t) = [1 \ 0 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

**Damping System Control:** To carry out a study and choice for the shock absorber control system and define its action, a control system that is dependent on the load value and the knee movement obtained at different moments of human gait is necessary.

The purpose of the control system is that it can read the input variables, transmitted load and knee position, we will focus in this work on the control part.

#### D. Results and Simulation

**Through State Space:** Once the mathematical model is obtained, we can develop the control system, which will allow us to regulate the behavior of the damping system. To analyze the system, the impulse function and the step function were used, as shown in Fig. 4. It is observed that the trend of the system is increasing, so the system is unstable and therefore it is necessary to include a controller.

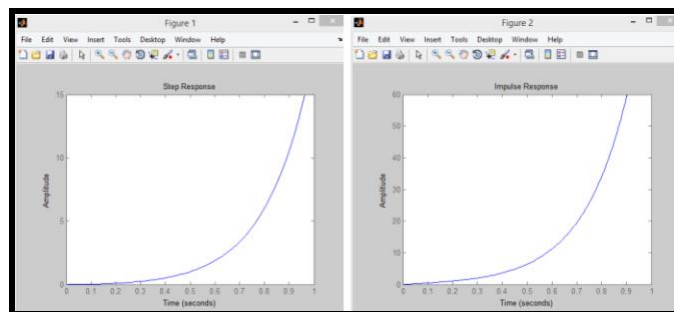


Fig. 4. System transfer function output.

In order to analyze the behavior of the entire system, the state variable form can be used, where the relevant matrices to the state model will be found, in discrete time and in continuous time, to manage the observability and controllability of the system, whose Values represent the gain, poles, and zeros of the system.

For this we will solve by means of complete feedback of states, for this type of control we see it in Fig. 5.

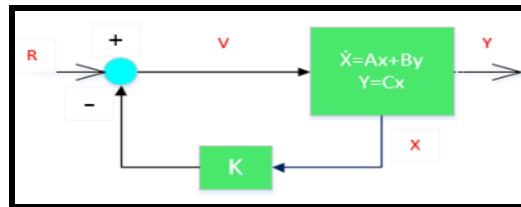


Fig.5. State feedback block diagram

In the input R the weight will remain at 1, when the Q matrix has been found, we can try to find the K matrix that has a favorable controller; Through Matlab programming the matrix K is found and the answer is also sketched. Fig. 6 shows the response of the system; as well as the displacement of the center of mass.

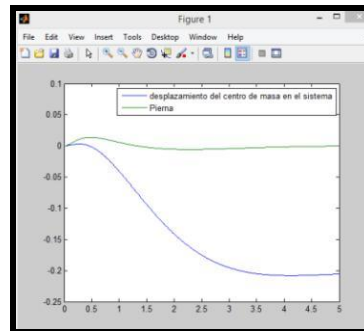


Fig. 6. Center of Mass and Leg Displacement Response.

The position of the center of mass and the angle of the knee of the system can be appreciated, it is observed that the over-peaks of the leg and the center of mass seem to be fine, but it is necessary to improve lowering the rise time of the center of mass and the settling time. In Fig. 7, after increasing the value of  $x$ , the rise and set times decrease, and the movement of the knee also decreases, resulting in the step response and the values of K.

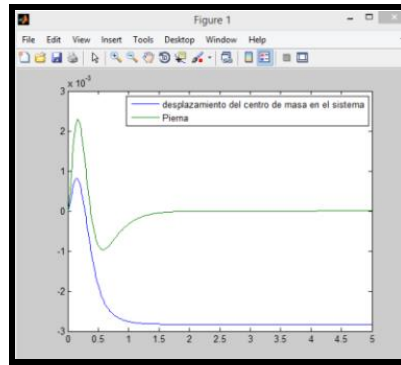


Fig. 7. Center of mass and leg varying X and Y.

Compared with the other system analysis methodologies, the steady state error must be removed, since the output will be fed back and a comparison is made with the reference input to be able to find an error, in this way an integer feedback controller of state are fed back all of each of the states; for this reason it is convenient to calculate the steady state value of the states; can be done by multiplying by the chosen gain K as well as using the new input value.

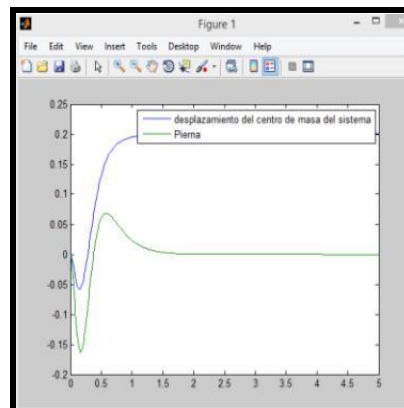


Fig. 8. Offset of the center of mass and leg with a chosen gain K and using the new value as a reference for the input.

It is observed that the response is optimal, which was achieved by examining complete state feedback, this is a valid estimate. The design requirements have been met, so no further testing is required.

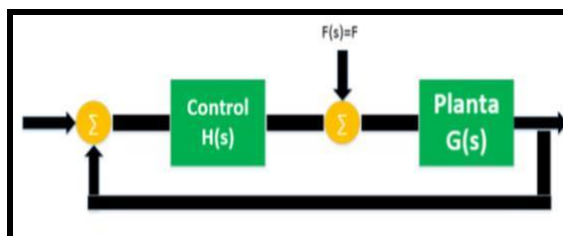


Fig. 9. Feedback control system.



**Through Frequency Response:** When the transfer function is introduced, the system is examined with a pulse by means of the Nyquist diagram. Nyquist is used because the system is unstable in open loop. The block diagram is shown in Fig. 9.

In the simulation, it was observed that a counterclockwise detour at -1 is required to obtain a stable closed-loop system. In Fig. 10 the uncompensated Nyquist diagram is observed, the gain of the transfer function system is examined, which is equal to one.

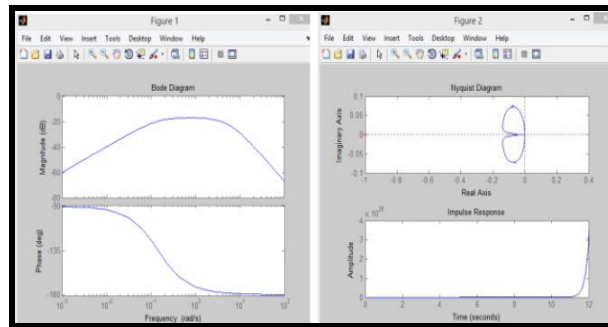


Fig. 10. Bode, Nyquist plot plots obtained.

In Fig. 11, it can be seen that when a zero was added it was not enough to change the direction of the bode diagram, since it is still clockwise, so what will be done now is to add another zero, and also some profit.

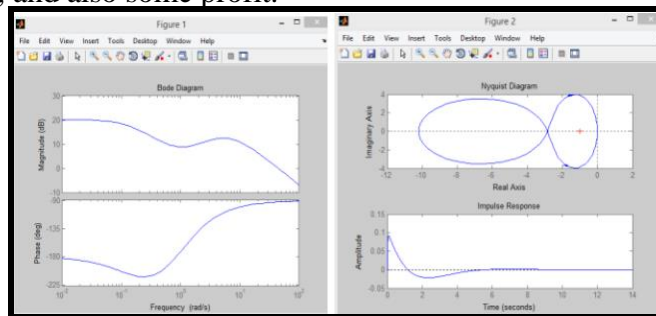


Fig. 11. Bode, Nyquist diagram plots, obtained with variation of parameters.

The system is stable, now it is only necessary to improve the response, for this the controller poles will be varied. Criterion, small poles, close to the origin, will modify the response at low frequencies, while large poles, far from the origin, will modify the response at high frequencies. Results in Fig. 12.

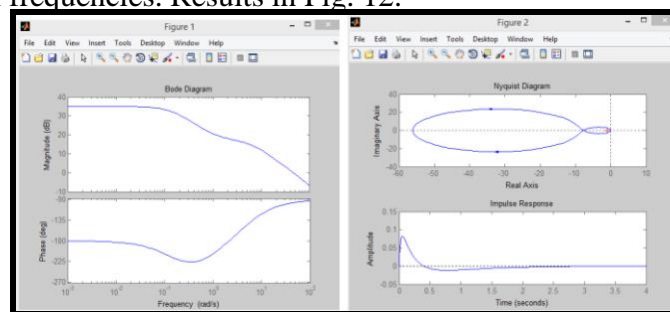


Fig. 12. Bode, Nyquist plot plots with values satisfying the controller.

After having made the changes made in the transfer function system, we can see how the answer has "flattened". In the bode diagram in the representation of the magnitude, the stability criterion is met since the resonance peak of the system is sufficient, since it is less than 1.5 and the bandwidth is large according to the time constant that it is small; It can also be seen that the system is compensated. It can also be said that another criterion that the system is stable is that the system is between 0dB to -180dB.

### III. CONCLUSIONS

The behavior of a prototype knee was simulated through SolidWork software, obtaining a structural analysis and also allowing us to see which were the most critical points, which are the fit of the knee with the socket and the holes that the shock absorber joins. to the knee.

The theory of state space and frequency response was used to design the controls of a prosthesis prototype, for that a mathematical modeling was done with which the control system used for the damping system with state equations and frequency response.

The worked model adjusts to the predefined situations (cost, control effort), these controllers essentially optimize the response of the system, approaching its output to that of a march without disability, since both the position of the center of mass of the system and the forces resemble their behavior with normal human gait.

The simulation of the designed control systems was carried out, obtaining favorable results.

A system that guarantees stability and mechanical firmness was achieved for the use of the transfemoral prosthesis.

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