Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Radial pulsation of a compact object in *d* dimensions

To cite this article: José D. V. Arbañil and Manuel Malheiro 2020 *J. Phys.: Conf. Ser.* **1558** 012003

Radial pulsation of a compact object in d dimensions

José D. V. Arbañil[1] and Manuel Malheiro[2]

[1]Departamento de Ciencias, Universidad Privada del Norte, Avenida el Sol 461 San Juan de Lurigancho, 15434 Lima, Peru

[2]Departamento de Física, Instituto Tecnológico de Aeronáutica, Centro Técnico Aeroespacial, 12228-900 São José dos Campos, São Paulo, Brazil

E-mail: [1]`jose.arbanil@upn.pe`, [2]`malheiro@ita.br`

Abstract. The influence of the extra dimensions on the equilibrium and radial pulsation of a compact object is investigated. For such purpose, we solve the stellar structure equations and radial pulsation equations, both modified from their original version to include the extra dimensions ($d \geq 4$) taking into account that spacetime outside the object is depicted by a Schwarzschild-Tangherlini metric. In addition, we consider that the pressure and the energy density are connected by a linear relation. Some properties of compact objects are analyzed, such as mass and period of the fundamental mode and their dependencies with the spacetime dimensions. We found that the maximum mass marks the begining of the instability, indicating that in a sequence of equilibrium configurations, the regions constitute by stable and unstable compact objects are distinguished by the relations $dM/d\rho_{cd} > 0$ and $dM/d\rho_{cd} < 0$, respectively.

1. Introduction

In the last years, as a result of a Kaluza-Klein theory [1], the idea that spacetime may have extra dimensions has become accepted. This inspired some authors to analyze some physical phenomena that arise in the study of compact objects, in both classical context and general relativity framework in higher-dimensional spacetime. To name a few, in the frame of Newtonian and Einstein gravity have been addresed the static equilibrium configurations of white dwarfs [2] and the equilibrium configuration of incompressible objects [3, 4] in d dimensions, respectively.

In Newtonian frame, inspired in the Chandrasekhar's seminal article [5], in [2] Chavanis find that fermions stars are unstable in higher dimensional spacetime, since the pressure of degenerate fermions that cannot counteract the gravitational force, leading to gravitational collapse. In general relativity framework, the static equilibrium configuration of compact object with constant central energy density in d dimensions [3, 4] is analyzed. Through the solution of stellar structure equations for a higher-dimensional spacetime, in these works, it is shown how the total mass depends of the dimensionality of the spacetime. Moreover, in [4] authors find that the dimensionality affects mass but not the fluid pressure.

Motivated in these works, in this article, in a higher-dimensional general relativity context, we study the equilibrium configurations and radial pulsations of objects composed by a fluid which a energy density and pressure following a linear relation. This is realized by integrating the stellar structure equations and radial pulsation equations. The dependence of some physical quantities with the spacetime are investigated, such as the total mass and period of the fundamental mode. The units $c = 1 = G_4$ are considered throughout the work, with c and G_4 being respectively the speed of light and four-dimensional gravitational constant.

2. General relativistic equations in d dimensions

2.1. Field equation, stress-energy tensor and the background spacetime

The compact objects properties in higher dimensional spacetime are investigated through the d-dimensional Einstein field equation for $d \geq 4$, which can be written into the form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{d-2}{d-3} S_{d-2} G_d T_{\mu\nu}\,, \tag{1}$$

with the quantities $R_{\mu\nu}$, R and $g_{\mu\nu}$ being respectively the Ricci tensor, Ricci scalar and the metric tensor. On the right-hand side of Eq. (1), $S_{d-2} = 2\pi^{(d-1)/2}/\Gamma((d-1)/2)$ represents the area of unitary sphere $\mathbf{S}^{d-2}$, with Γ being the usual gamma function, G_d the universal constant which in four dimensions corresponds to the Newton's gravitational constant G_4 and the factor $(d-2)\, G_d S_{d-2}/(d-3)$ corresponds to the $8\pi G_4$ term in four dimensions (see [6]). $T_{\mu\nu}$ is the stress-energy tensor of a perfect fluid. It is given by

$$T_{\mu\nu} = (\rho_0 + p_0)\, U_\mu U_\nu + p_0\, g_{\mu\nu}, \tag{2}$$

with ρ_0 being the energy density, p_0 the pressure of the fluid and U_μ the velocity of the fluid in the d-dimensional spacetime. In the definitions previously used, the Greek indexes μ, ν, etc., run from 0 to $d-1$, where 0 represents the time, and the other $d-1$ coordinates are spacelike.

We consider that the spherically symmetric distribution of the static fluid in the object is described by the d-dimensional spacetime:

$$ds^2 = -e^{\nu_0(t,r)}\, dt^2 + e^{\lambda_0(t,r)}\, dr^2 + r^2 \sum_{i=1}^{d-2} \left(\prod_{j=1}^{i-1} \sin^2\theta_j \right) d\theta_i^2. \tag{3}$$

It is important to say that the metric functions $\nu_0(t,r)$ and $\lambda_0(t,r)$ and the fluid variables $p_0(t,r)$, $\rho_0(t,r)$ depend on the coordinates t and r. To analyze the stability against small radial disturbances, both spacetime and fluid variables are perturbed. Following [7], we decompose the variables depended on the coordinate t and r of the form:

$$f_0(t,r) = f(r) + \delta f(t,r), \tag{4}$$

where $f(r)$ sets the unperturbed spacetime and physical quantities which depend on the radial coordinate. On the other hand, $\delta f(t,r)$ is the Eulerian perturbation. It depends on t and r.

2.2. Stellar structure equations

The field equation (1) and the line element (3), in the unperturbed system $\delta f(t,r) = 0$, lead to derive the following ensemble relations:

$$\frac{dm}{dr} = S_{d-2}\rho_d\, r^{d-2}, \tag{5}$$

$$\frac{dp_d}{dr} = -(p_d + \rho_d) G_d \left[\frac{S_{d-2} p_d\, r}{(d-3)} + \frac{m}{r^{d-2}} \right] e^{\lambda}, \tag{6}$$

$$\frac{d\nu}{dr} = -\frac{2}{(p_d + \rho_d)} \frac{dp_d}{dr}, \tag{7}$$

with the metric function $e^{-\lambda} = \left(1 - \frac{2m\, G_d}{(d-3) r^{d-3}}\right)$. The function $m\, G_d/(d-3)$ depicts the gravitational mass in d-dimensional spacetime within the hypersphere radius r. Equation (6) is TOV equation [8] modified from its standard form to consider the spacetime dimension [4]. Equations (5)-(7) are known like the stellar structure equations.

To obtain static equilibrium solutions, the stellar structure equations, Eqs. (5)-(7), are integrated from the center $r = 0$ to the star's surface $r = R$. In $r = 0$, it is considered $m(0) = 0$, $\lambda(0) = 0$, $\nu(0) = \nu_c$, $p_d(0)\,G_d = p_{cd}\,G_d$, and $\rho_d(0)\,G_d = \rho_{cd}\,G_d$. The constants $p_{cd}\,G_d$ and $\rho_{cd}\,G_d$ depict respectively central pressure and the central energy density. On the other hand, the star's surface is reached when $p_d(R)\,G_d = 0$. At this point, the interior metric matches smoothly to the exterior Schwarzschild-Tangherlini metric [9], where the interior and the exterior potential metrics fulfill the equality $e^{\nu(R)} = e^{-\lambda(R)} = 1 - \frac{2M\,G_d}{(d-3)r^{d-3}}$, where $M\,G_d/(d-3)$ is the total mass.

2.3. Radial stability equations
To derive the equations that govern the stability against small radial perturbations, following [7], both potential metric and fluid variables are divided into the form established in Eq. (4). These variables are replaced into the Einstein equation components and the stress-energy tensor while retaining only the first-order terms. For a system in a d-dimensional spacetime, the oscillation equations can be placed into the form [10]:

$$\frac{d\xi}{dr} = \frac{\xi}{2}\frac{d\nu}{dr} - \frac{1}{r}\left((d-1)\xi + \frac{\Delta p_d}{p_d\Gamma_1}\right), \tag{8}$$

$$\frac{d\Delta p_d}{dr} = \frac{\xi r e^{\lambda}}{e^{\nu}}(p_d+\rho_d)\omega^2 + \frac{(p_d+\rho_d)r\xi}{4}\left(\frac{d\nu}{dr}\right)^2 - 2S_{d-2}G_d(p_d+\rho_d)e^{\lambda}r\xi\left(\frac{p_d}{d-3}\right)$$
$$-\left(S_{d-2}G_d\frac{re^{\lambda}(p_d+\rho_d)}{d-3} + \frac{1}{2}\frac{d\nu}{dr}\right)\Delta p_d - 2(d-2)\xi\frac{dp_d}{dr}, \tag{9}$$

where ξ denotes the relative radial displacement, Δp_d depicts the Lagrangian perturbation of pressure, $\Gamma_1 = \left(\frac{p_d+\rho_d}{p_d}\right)\frac{dp_d}{d\rho_d}$ represents the adiabatic index and ω the eigenfrequency of oscillation.

To study the radial stability, Eqs. (8) and (9) are integrated from the center toward the surface of the object. In the center is demanded $\Delta p_d\,G_d = -(d-1)\,(\xi\Gamma_1\,p_d\,G_d)_{\rm center}$. At this point, it is required $\xi(r=0) = 1$. In turn, at the surface of the hypersphere, we have $(\Delta p_d\,G_d)_{\rm surface} = 0$.

2.4. Equation of state
For the fluid, we consider that the pressure p_d and energy density ρ_d are related of the form:

$$\rho(r) = (d-1)p(r) + d\,\mathcal{B}_d, \tag{10}$$

with $\mathcal{B}_d$ being a constant. In this work, we consider $d\,\mathcal{B}_d\,G_d = 240\,[\mathrm{MeV/fm^3}]$.

3. Results
The changing of the total mass versus the central energy density $\rho_{cd}\,G_d$ is presented on the left-hand side of panel Fig. 1 for four spacetime dimensions. The energy densities considered are in the interval $250 \leq \rho_{cd}\,G_d \leq 5000\,[\mathrm{MeV/fm^3}]$. At this range, we note the increment of the total mass with the central energy density until reach the maximum mass point $M_{\rm max}/M_\odot$, marked by a triangle, hereafter, the mass decreases with the grow of the central energy density. It is important to highlight that, in all cases, we obtain $M_{\rm max}/M_\odot$ at the zero eigenfrequency of oscillation $\omega = 0$ ($\tau_{n=0} \to \infty$). From this we conclude that, in the sequence of equilibrium configuration, the peak of maximum total mass marks the onset of the instability; in other words, regions constituted by stable a unstable equilibrium configurations can be identify by the conditions $dM/d\rho_{cd} > 0$ and $dM/d\rho_{cd} < 0$. In turn, the dependence of the period of the fundamental mode $\tau_{n=0}$ with the total mass is shown on the right-hand side of panel Fig. 1 for some spacetime dimensions. In all cases, we see the monotonic growth of the period of the fundamental mode $\tau_{n=0}$ with the total mass $M/M_\odot$. From this we understand that more massive objects demand more time to reach radial stability.

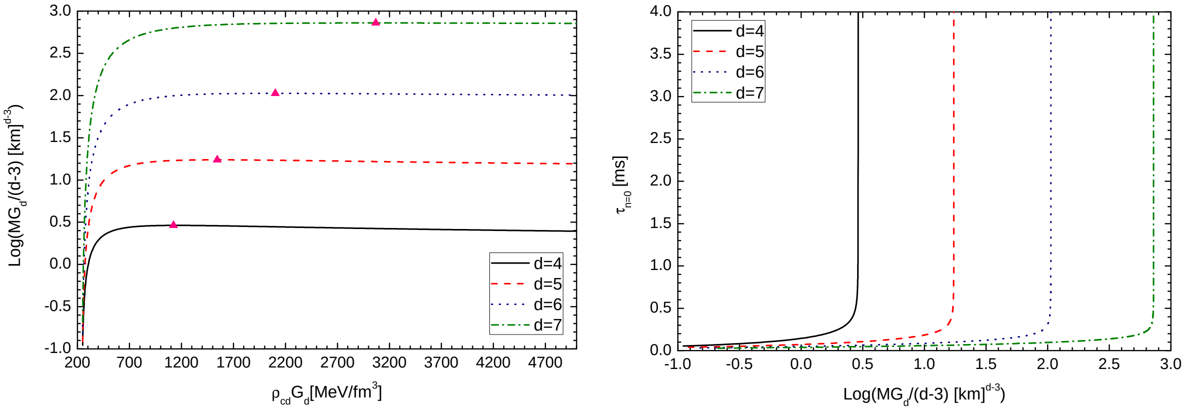

Figure 1. Left panel: Total mass against the central energy density. Right panel: The period of the fundamental oscillation mode versus the total mass. In both figures, four spacetime dimensions are considered.

4. Conclusions

In this work, we investigated the equilibrium configurations and radial pulsations of compact objects made of a fluid which follows a linear relation between the energy density and fluid pressure in d dimensions. The configurations under analyzis have hyperspherically symmetry and are connected to the exterior Schwarzschild-Tangherlini spacetime. The hydrostatic equilibrium equations and radial pulsation equations were solved for some $\rho_{cd} G_d$ and d.

Through the fourth-order Runge-Kutta method implemented with the shooting method, we found that the maximum mass point $M_{\rm max}/M_\odot$ and the zero eigenfrequency of the fundamental mode ω ($\tau_{n=0} \to \infty$) are derived with the same central energy density value. This indicates that $M_{\rm max}/M_\odot$ marks the begining of the instability against small radial perturbations. I.e., in sequence of equilibrium configurations, the conditions are $dM/d\rho_{cd} > 0$ and $dM/d\rho_{cd} < 0$ are necessary and sufficient to recognize regions composed by stable and unstable objects. Moreover, in each dimension d, we note that the period of the fundamental mode $\tau_{n=0}$ grows with $M/M_\odot$. This indicate that more massive compact objects require more time to attain the radial stability.

Acknowledgments

We thank Fundação de Amparo à Pesquisa do Estado de São Paulo-FAPESP, under the thematic project 2013/26258-4, for the grant.

References

[1] Kaluza T 2018 *Int. J. Mode. Phys. D* **27** 1870001; Klein O 1926 *Zeitschrift für Physik* **37** 895
[2] Chavanis P -H 2007 *Phys. Rev. D* **76**, 023004
[3] Harko T and Mak M 2000 *J. Math. Phys.* **41**, 4752
[4] Ponce de Leon J and Cruz N 2000 *Gen. Relativ. Gravit.* **32**, 1207
[5] Chandrasekhar S 1935 *Mon. Not. R. Astron. Soc.* **95**, 207
[6] Lemos J P S and Zanchin V T 2008 *Phys. Rev. D* **77**, 064003
[7] Chandrasekhar S 1964 *Astrophys. J.* **140**, 417
[8] Tolman R C 1939 *Phys. Rev. D* **55**, 364; Oppenheimer J R and Volkoff G 1939 *Phys. Rev. D* **55**, 374
[9] Tangherlini F L 1963 *Nuovo Cimento* **27**, 636; Lemos J P S and Zanchin V T 2009 *Phys. Rev. D* **80**, 024010
[10] Arbañil J D V, Carvalho G A, Lobato R V, Marinho Jr R M, and Malheiro M 2019 *Phys. Rev. D* **100**, 024035